

Citation for published version:

Huang, Y, Huang, GQ & Newman, ST 2011, 'Coordinating pricing and inventory decisions in a multi-level supply chain: A game-theoretic approach', *Transportation Research Part E: Logistics and Transportation Review*, vol. 47, no. 2, pp. 115-129. <https://doi.org/10.1016/j.tre.2010.09.011>

DOI:

[10.1016/j.tre.2010.09.011](https://doi.org/10.1016/j.tre.2010.09.011)

Publication date:

2011

[Link to publication](https://doi.org/10.1016/j.tre.2010.09.011)

University of Bath

Alternative formats

If you require this document in an alternative format, please contact:
openaccess@bath.ac.uk

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Coordinating Pricing and Inventory Decisions in a Multi-Level Supply Chain: A Game-Theoretic Approach

Yun HUANG ^a, George Q. HUANG ^{a,*}, Stephen T. Newman ^b

^a Department of Industrial and Manufacturing Systems Engineering, University of Hong Kong, Hong Kong, PR China

^b Department of Mechanical Engineering, University of Bath, Bath, UK

Abstract

This paper concerns coordination of enterprise decisions such as suppliers and components selection, pricing and inventory in a multi-level supply chain composed of multiple suppliers, a single manufacturer and multiple retailers. The problem is modeled as a three-level dynamic non-cooperative game. Analytical and computational methods are developed to determine the Nash equilibrium of the game. Finally, a numerical study in computer industry is conducted to understand the influence of the market scale parameter and the components selection strategy on the optimal decisions and profits of the supply chain as well as its constituent members. Several research findings have been obtained.

Keywords: Multi-level supply chain, dynamic non-cooperative game, Nash equilibrium, product family design, pricing, inventory.

1 Introduction

A supply chain consists of geographically distributed and administratively decentralized business partners. In such a decentralized supply chain, decisions of individual partners are often not coordinated with each other. Their local objectives are often inconsistent with those of the entire system objectives. As a result, the supply chain becomes less competitive (Porter 1985). Many firms and researchers focus on coordinating pricing and inventory decisions to optimize the entire system and improve the efficiency of both the supply chain and individual firms (Weng, 1995; Chan et al., 2004).

Typically, a supply chain involves a variety of multiple products that are related to each other through common features. The levels of product variety offered by supply chains have demonstrated increasing trends (Macduffie et al., 1996). The product family design and platform products development have been widely used to increase variety, shorten lead times, and reduce costs (Simpson, 2005). The research in this paper has been motivated to integrate the product family design and platform products development into the pricing and inventory decisions to coordinate a decentralized supply chain.

This paper focuses on joint decision-making about the selection of suppliers and components of a product family (Meyer and Utterback, 1993). The emphasis is placed upon the coordination of suppliers and components selection, pricing and inventory decisions (CSCSPI) in a multi-level supply chain consisting of multiple suppliers, one manufacturer and multiple retailers. The manufacturer purchases optional components of certain functionality from his alternative suppliers to produce a set of platform products to meet the requirements from the retailers in different markets. Each supplier faces the problem to make decisions on the prices for the components he sells to maximize his net profit. The manufacturer has to determine the setup time interval for production, the wholesale prices, and the suppliers and

* Corresponding author: Tel.: (00852) 2859 2591

E-mail addresses: huangyun@hku.hk (Y. Huang), gqhuang@hku.hk (G.Q. Huang), s.t.newman@bath.ac.uk (S.T. Newman)

components selection decisions to maximize his net profit. The retailers' problem will focus on the replenishment cycles and retail prices for the products.

We describe CSCSPI problem as a three-level dynamic non-cooperative game with respect to the overall supply chain. The suppliers formulate the bottom-level non-cooperative simultaneous sub-game and at the same time as a whole play the middle-level non-cooperative simultaneous sub-game with the manufacturer. The suppliers and the manufacturer also being a group formulate the top-level non-cooperative simultaneous whole game with the retailers. Once the whole game settles an equilibrium solution, none of the any chain members is able to improve its payoff (i.e. profits) by acting unilaterally without degrading the performance of other players. We propose both analytical and computational methods to obtain the Nash equilibrium of this game.

The game model and the proposed solution algorithm constitute a powerful decision support for solving the CSCSPI problem. Its use is demonstrated and tested through a numerical example. The impacts of the market scale parameter and components selection on the decisions and profits of all the chain members are also investigated.

This paper is structured as follows. The next section presents a brief review of the literature related to pricing and inventory coordination, product family design, Game Theory for supply chain coordination. In Section 3, we give the problem description and some notations. We formulate the mathematical model of the CSCSPI problem in Section 4. Section 5 proposes the analytical and computational methods used to solve the CSCSPI problem in Section 4. In Section 6, a numerical study and the influence of market scale parameter and the components selection strategy have been presented. Finally, this paper concludes in Section 7 with some limitations and suggestions for further work.

2 Literature review

Pricing, inventory decisions, and product family design and platform products development, have been extensively studied in supply chain coordination. Although the three areas are closely interrelated with each other, they are rarely been studied in an integrated, systematic manner. Recently, Game Theory (GT) has also been applied to analyze supply chain coordination problem. This section will briefly review a few representative works related to this research.

2.1 Coordination of pricing and inventory decisions

Coordinating pricing and inventory decisions of supply chain has been studied by researchers for more fifty years. Whitin (1955) shows that the retailer could obtain greater profits when coordinating the price and order quantity decisions. Kunreuther and Richard (1971) find the same results for the inter-department coordination in a manufacturer and a retailer. Based on their work, Tersine and Price (1981), Arcelus and Srinivasan (1987), Ardalan (1991), Martin (1994), and Abad (2003) draw the same conclusions in various circumstances. Kim and Lee (1998) examine the joint pricing and lot sizing problem for a profit-maximizing firm facing constant and price dependent demand with both fixed and variable capacity. Weng and Wong (1993) and Weng (1997) propose a model of seller-buyer relationship and confirm that coordinated decisions on pricing and inventory benefit both the individual chain members and the entire system. Boyaci and Gallego (2002) analyze the problem of coordinating pricing and inventory replenishment policies in a supply chain consisting of a wholesaler, one or more geographically dispersed retailers. They show that optimally coordinated policy could be implemented cooperatively by an inventory-consignment agreement. Prafulla et al. (2006) present a set of models of

coordination for pricing and order quantity decisions in a one manufacturer and one retailer supply chain. They also discuss the advantages and disadvantages of various coordination possibilities. These studies on the coordination of pricing and inventory decisions problem focus on individual entities or two-stage channels.

2.2 Product family design and platform strategy

Thonemann and Bradley (2002) study the impact of product variety on supply chain performance. Kohli and Sukumar (1990) deal with a joint problem of designing a set of optional products to maximize the manufacturer's profit. Various models have been derived for designing the product family instead of a single product to reduce the cost concerns of increased product variety, as Yano and Dobson (1998) reviewed. Chakravarty and Baum (1992) formulate a product family model incorporating process selection and use it to illustrate the interactions with some marketing and manufacturing variables. Saurabh and Krishnan (1999) examine the reduction in complexity of a product family by product design. Park and Simpson (2005) show the benefits of different resource sharing methods related to product family design.

Meyer and Lehnerd (1997) define product platform as the shared common components, product structure and manufacturing assets by a product family. The advantages to design products based on platforms are demonstrated in component demand patterns, work center load, work-in-progress inventory and delivery performance (Collier, 1981). Sanderson and Mustafa (1995) show the impacts of using a platform strategy on the amount of product variety offered by a company. There are also a few model-based approaches to creating products based on platforms. Krishnan et al. (1998) propose a way to obtain an optimal platform-based family based on a network model for products that can be measured along a single performance index that may increase with time.

2.3 GT for supply chain coordination

Game Theory has been used as an alternative to study supply chain coordination problem as reviewed by Cachon and Netessine (2004). The quantity discount games, inventory games have also been studied extensively. Weng (1995) study a supply chain with one manufacturer and multiple identical retailers. He shows that the Stackelberg game guaranteed perfect coordination considering quantity discounts and franchise fees. Parlar and Wang (1994), address the quantity discount problem of how a supplier should design his discount pricing policy to maximize his profit as well as make the buyers better off using game theory approach. Their work is extended by Wang and Wu (2000) to the case with heterogeneous buyers. Lariviere and Porteus (2001) and Slikker et al. (2005) study the news vendor problem by Game theory approach. Moyaux et al. (2004) analyzes the informing sharing and bullwhip effect in a supply chain through a normal form game.

In contrast with quantity games and inventory games, GT applications to product family design are very limited. Bakos and Brynjolfsson (1993) develop a game model to study the optimal number of suppliers for a buyer. Huang et al. (2007) optimize the configuration of a set of platform products and the associated supply chain using a three-move dynamic game-theoretic approach.

Although GT is employed to study supply chain coordination problem in above research, each of them focusing on only one aspect and does not integrate quantity discount pricing policy, inventory decisions and product family design. Yu et al. (2006) simultaneously consider pricing and order intervals as decisions variables using Stackelberg game in a supply chain with one manufacturer and multiple retailers. Esmaeili et al. (2009) proposed several

game models of seller-buyer relationship to optimize pricing and lot sizing decisions. Game-theoretic approaches are employed to coordinate pricing and inventory policies in these studies, but none of the authors involves product family design or platform product development in their coordination problems. Zhang (2006) integrates platform product development with supply chain configuration, marketing and inventory decisions using a dynamic game. However, her research focuses on the two-level supply chain.

3 Problem description and some notations

In this section, we describe the CSCSPI problem using an illustrative supply chain adapted from Grave and Willems (2005) and Zhang (2006) in Fig. 1. This application case is concerned with a three-echelon supply chain involving multiple suppliers, one single manufacturer and multiple retailers. The manufacturer, indicated by m , designs and customizes a set of platform products for retailers ($r_l, l=1,2,\dots,L$) in different independent market segments. Each retailer is served by one product customized from the product platform. In Fig. 1, the manufacturer focuses on two computer platform products, involving Notebook A and Notebook B, and sells them to the retailers in two market regions respectively, namely Europe (EU) and North America (NA).

The architecture for the product platform consists of a series of different functionality elements. Once the architecture is finalized, components are designed and selected to offer certain functionality. The components offering different levels of the same functionality are grouped together in the substitutable component set (SCS), indexed by $i=1,2,\dots,I$. Suppose that the components within the same SCS can be ranked in order of decreasing functionality and higher functionality components can substitute ones with lower functionality completely, but not vice versa. Thus, in this paper, components selection for the manufacturer is to decide whether to choose higher functionality components to replace lower ones fixed a priori and what higher functionality components to choose.

Let N_i be the number of components in SCS i . L_{ij} is used to denote the component which is the j^{th} element in the i^{th} SCS, where $i=1,2,\dots,I$ and $j=1,2,\dots,N_i$. In Fig.1, the two platform products share the same architecture, with SCSs, processor, LCD display, memory, hard drive, miscellaneous components and metal housing, in sequence. Among them, processor and LCD display have several component options, ranked in order of decreasing functionality. For instance, in processor SCS, Intel Pentium Dual-core has been fixed for Notebook B, but the manufacturer could select higher functionality Intel Core 2Duo to replace it. For those SCSs, which involve one component only, we do not distinguish SCS or component for them. For example, for the memory SCS and memory component, we use the same sign to denote, as Fig. 1 shows.

All the components are purchased from a fixed number of alternative suppliers ($s_v, v=1,2,\dots,V$). We assume that each supplier's capacity is enough to satisfy the needs of the manufacturer. For the components in Fig. 1, there are 6 alternative suppliers. Fig. 1 also shows the relationship between the suppliers and the components they provided. For example, supplier 1 provides Intel Core 2Duo processor and SXGA+ display.

The suppliers, the manufacturer and the retailers are assumed to be rational decision makers and have equal market power. An immediate question faced by each supplier is how to determine the prices for the components he sells to maximize his net profit. The manufacturer will have to determine the setup time interval for production, the wholesale prices, even the suppliers and components selection decisions to maximize his net profit. The retailers' problem will focus on their replenishment cycles and retail prices for the products. Thus,

under this supply chain circumstance, these competing non-cooperative suppliers reach an equilibrium on their pricing decisions and as a whole negotiate with the manufacturer on their pricing, inventory and suppliers and components selection decisions to maximize their own profits. Negotiation will also be conducted between the manufacturer and the retailers on their pricing and inventory decisions. After the suppliers, the manufacturer and the retailers reach an agreement, the manufacturer will purchase these components from the suppliers to produce different products for the retailers.

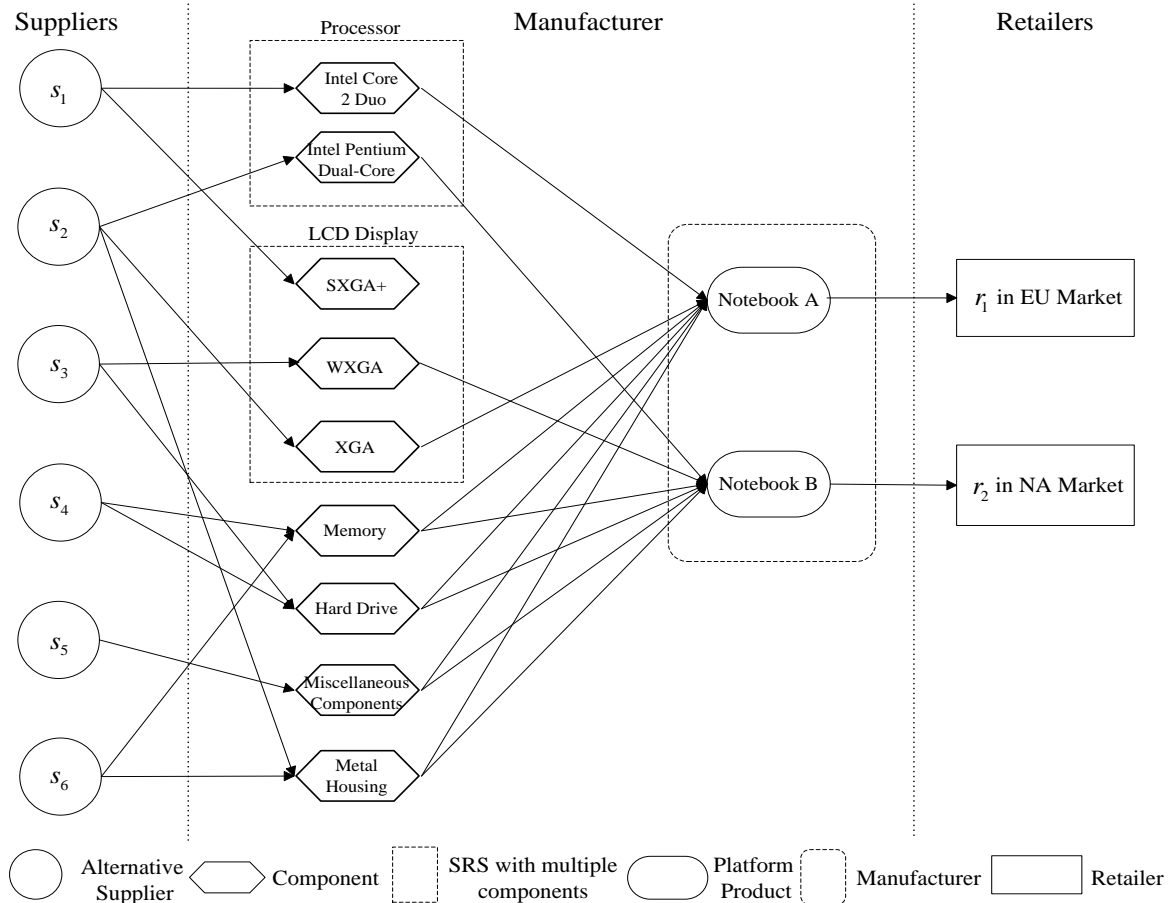


Fig. 1. Coordination of suppliers and components selection, pricing and inventory decisions (CSCSPI): an illustration

We then give some other assumptions used for building the mathematical model in the next section.

- (1) The integer multipliers mechanism (Moutaz, 2003) for replenishment is adopted between the manufacturer and the retailers. That is, the manufacturer's setup time interval is integer multipliers of the replenishment cycle time of the retailers.
- (2) The suppliers and the manufacturer use vendor managed inventory (VMI) system (Simchi-Livi et al., 2000; Tyan and Wee, 2003) to replenish their components. This inventory system has been adopted by some industries for years (e.g. Wal-Mart, Procter&Gamble (P&G), Dell, etc.). Under this assumption, the components' inventories for the manufacturer and the suppliers are at the side of their upstream suppliers and their corresponding inventory holding costs are also borne by their upstream suppliers (Birendra and Srinivasan, 2004).
- (3) Single sourcing strategy (Tullous and Utrecht, 1992) is adopted between supplies and manufacturer. Thus, the manufacturer purchases one type of component from only one supplier.

- (4) Either all or none of the demand of a component is replaced by the lower order component in the same SCS.
- (5) Shortage are not permitted, hence the annual production capacity is greater than or equal to the total annual market demand (Esmaeili, 2008).

4 The mathematical model

4.1 Game scheme

We model the CSCSPI problem as a three-level dynamic non-cooperative game with $V+L+1$ players, i.e., V suppliers, one manufacturer and L retailers. Each supplier controls his strategy set X_{s_v} ($v=1,2,...,V$) to maximize his payoff function Π_{s_v} . A strategy $x_{s_v} \in X_{s_v}$ includes pricing decisions for the components supplied by the supplier. The manufacturer controls the strategy set X_m to maximize his payoff function Π_m . His strategy $x_m \in X_m$ consists of setup time interval, profit margin for each product, and suppliers and components selection decisions. Each retailer controls his strategy set X_{r_l} , whose strategy $x_{r_l} \in X_{r_l}$ is composed of replenishment decisions and profit margin, to maximize his payoff function Π_{r_l} .

In our game framework, the suppliers formulate the bottom-level non-cooperative simultaneous sub-game (called SS game for simplicity of reference) and at the same time as a group formulate the middle-level non-cooperative simultaneous sub-game (called MS game for reference) with the manufacturer. Also, the top-level non-cooperative simultaneous whole game (called RMS game for reference) is played between the suppliers and the manufacturer as a whole sector and all the retailers.

According to Basar and Olsder (1982), the proposed RMS game is, therefore, a dynamic game where the retailers' strategies x_{r_l} ($l=1,2,...,L$) in RMS game affect the manufacturer's strategy x_m and the suppliers' strategies x_{s_v} ($v=1,2,...,V$) in MS game, and the decisions from MS game also affect those decisions in the RMS game. At the same time, the manufacturer's strategies in MS game also affect the suppliers' strategies x_{s_v} in SS game and vice versa. Through the dynamic interactions between RMS game, MS game and SS game, individual suppliers, manufacturer and retailers could determine optimal decisions to maximize their own payoffs. We can see this game is partially dynamic, because it is not a sequential game between the suppliers, the manufacturer and the retailers. Each player acts only once and move simultaneously (i.e. statically) in the game. That is for the suppliers, the manufacturer and the retailers, each of them determines their optimal decisions simultaneously. The whole RMS game in this paper is also a non-cooperative game with complete information where each player knows the other players' strategy sets but they are not involved in any binding agreements. Fig. 2 shows the structure of the RMS game.

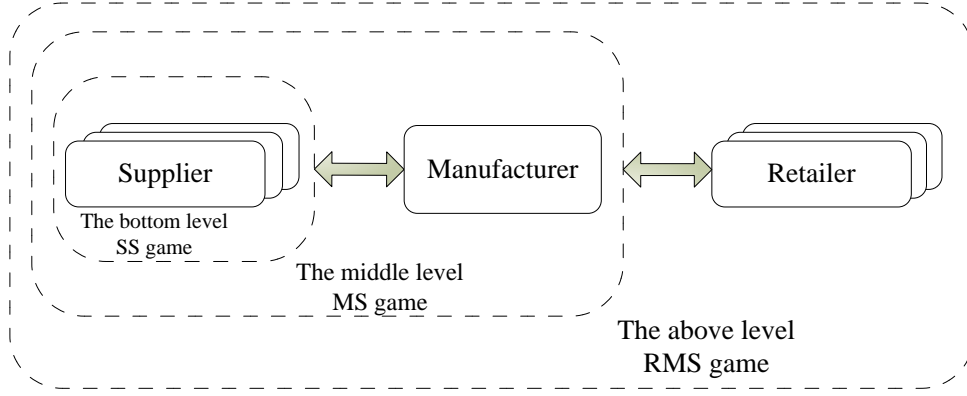


Fig. 2. Three-level dynamic non-cooperative game structure

4.2 The mathematical model

4.2.1 The retailers' model

We first consider the objective (payoff) function Π_{r_l} ($l=1, 2, \dots, L$) for the retailers. The retailer's objective is to maximize his net profit by optimizing his strategy x_{r_l} , including replenishment decision and profit margin. The relevant parameters and variables of the retailer l are designed in Table 1.

Table 1. Parameters and variables for retailers

D_l :	Retailer l 's annual demand
a_l :	A constant in the demand function of retailer l , which represents his market scale
b_l :	Coefficient of the product's demand elasticity for retailer l
h_l :	Retailer l 's holding cost per unit of product inventory
O_l :	Ordering processing cost for retailer l per order of product l
ρ_l :	Retailer l 's annual fixed costs for the facilities and organization to carry this product
p_l :	Retail price charged to the customer by retailer l
k_l :	Decision variable, the integer divisor used to determine the replenishment cycle of retailer l
g_l :	Decision variable, retailer l 's profit margin

The demand at retailer l is almost invariably a downward sloping and convex function with respect to the retail price in real world. We employ the linear demand function in McGuire and Staelin (1983) and Jeuland and Shugan (1988):

$$D_l(p_l) = a_l - b_l p_l, \quad (1)$$

where a_l is a constant and b_l is coefficient of the product's demand elasticity.

As indicated in the first point of the assumption in section 3, the integer multipliers mechanism is employed between the manufacturer and the retailers. Since the setup time interval for the manufacturer is assumed to be T , the replenishment cycle for retailer l is T/k_l . k_l should be a positive integer. Thus, the annual holding cost is $h_l T D_l / 2 k_l$ (see Fig. 3(a)) and the ordering process cost is $O_l k_l / T$.

The retailer l faces the holding cost, the ordering cost and an annual fixed cost. Therefore, the retailer l 's objective function is given by the following equation:

$$\max_{k_l, g_l} \Pi_l = g_l D_l - \frac{TD_l}{2k_l} h_l - \frac{O_l k_l}{T} - \rho_l \quad (2)$$

Subject to

$$k_l \in \{1, 2, 3, \dots\}, \quad (3)$$

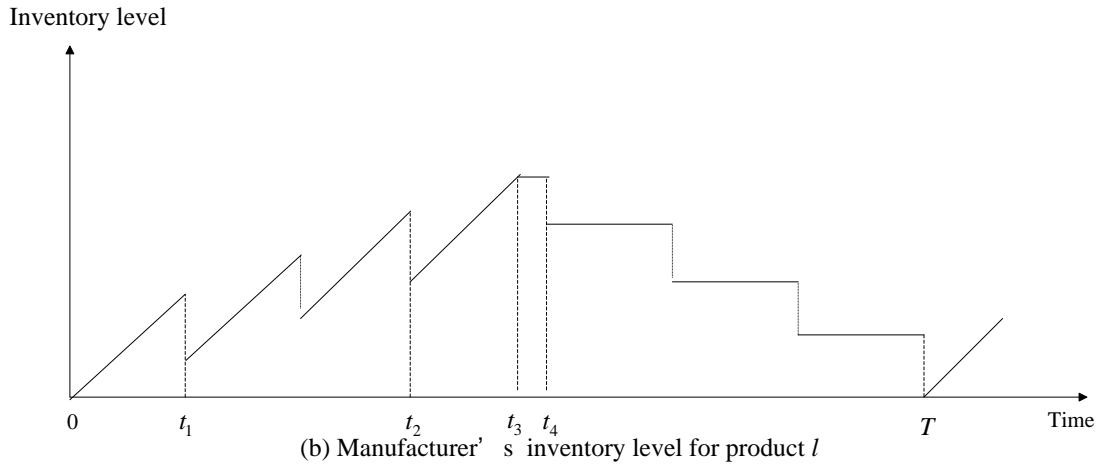
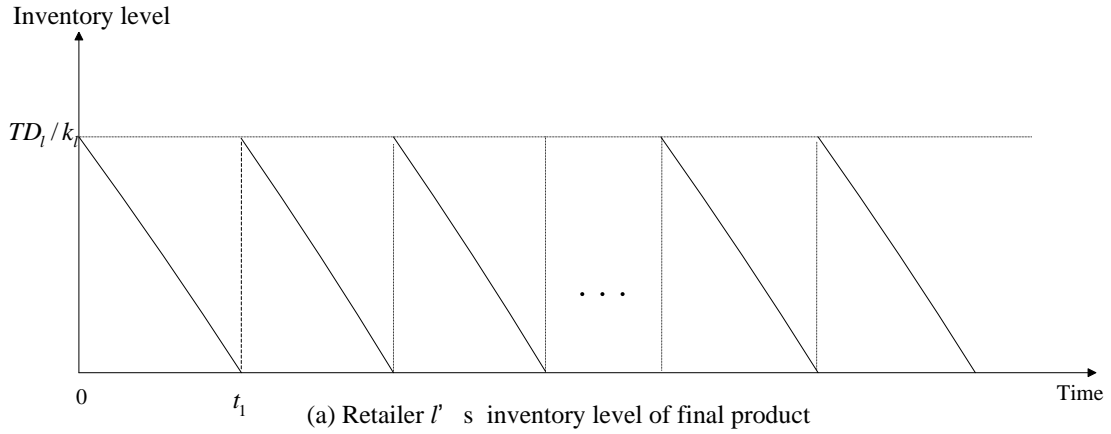
$$g_l = p_l - \omega_{m_l}, \quad (4)$$

$$D_l = a_l - b_l p_l, \quad (5)$$

$$g_l \geq 0, \quad (6)$$

$$0 \leq D_l \leq P_l. \quad (7)$$

Constraint (3) gives the value of the divisor used to determine the retailer l 's replenishment cycle time. Constraint (4) indicates the relationship between the prices (the retail price p_l and the wholesale price ω_{m_l}) and retailer l 's profit margin. Constraint (6) ensures that the value of g_l is nonnegative. Constraint (7) gives the bounds of the annual demand, which cannot exceed the annual production capacity P_l of the product.



$$t_1 = T / k_l; t_2 = (n_l - 1) T / k_l; t_3 = TD_l / P_l; t_4 = n_l T / k_l$$

Fig. 3. Inventory of retailer and manufacturer

4.2.2 The manufacturer's model

The manufacturer's objective is to determine his decision vector x_m , composed of the setup time interval for production, the profit margins for all the products and the selection decision of suppliers and components, to maximize his net profit. The relevant parameters and

decisions variables are shown in Table 2.

Table 2. Parameters and variables for manufacturer

P_l : Annual production capacity for product l
c_{m_l} : Production cost per unit product l
h_{m_l} : Manufacturer's holding costs per unit of product l inventory
S : Setup cost per production
F_{ij} : Fixed cost of using component L_{ij}
u_{ijl} : Predefined usage amount of unit component L_{ij} per unit product l
δ_v : The fixed cost of using supplier v , covering supplier certification, contract setup, etc.
γ : Annual fixed costs for the facilities and organization for the production of the products
ω_{m_l} : Wholesale price charged to retailer l
T : Decision variable, manufacturer's setup time interval
q_l : Decision variable, manufacturer's profit margin for product l
τ_{ijk} : Binary decision variable to indicate whether component L_{ij} has been used to replace L_{ik}
σ_v : Binary decision variable to indicate whether supplier v is used
t_{vij} : Binary decision variable to indicate whether component L_{ij} is supplied by supplier v
z_{ij} : Binary decision variable to indicate whether component L_{ij} is used

The manufacturer faces the costs of components used, production cost, fixed costs to contract with the suppliers and using components, holding costs for the products, setup cost, and an annual fixed cost. According to the given VMI policy, the manufacturer does not have to pay for the inventory cost for components. The behavior of the inventory level for the product for the manufacturer is illustrated as Fig. 3(b). The production time of product l is TD_l / P_l . There exists an integer n_l satisfying $n_l T / k_l \geq TD_l / P_l \geq (n_l - 1) T / k_l$. The inventories of product l from 0 to $(n_l - 1) T / k_l$, from $(n_l - 1) T / k_l$ to $n_l T / k_l$, and from $n_l T / k_l$ to T are::

$$\begin{aligned}
 I_{1,l} &= \frac{1}{2} (P_l - D_l) (n_l - 2) (n_l - 1) T^2 / k_l^2 + \frac{1}{2} (n_l - 1) P_l T^2 / k_l^2, \\
 I_{2,l} &= (P_l - D_l) (n_l - 1) T^2 / k_l^2 + \frac{1}{2} T \left[\left(1 - n_l^2 \right) P_l / k_l + 2 n_l D_l / k_l - \frac{D_l^2}{P_l} \right], \\
 I_{3,l} &= \frac{1}{2} (1 - n_l / k_l) \left(1 + (1 - n_l) / k_l \right) D_l T^2.
 \end{aligned}$$

The annual inventory for product l 's is $(I_{1,l} + I_{2,l} + I_{3,l}) / T$ or given by $TD_l (1 + 1/k_l - D_l / P_l) / 2$ through algebraic manipulations (see Lu 1995). The setup cost S occurs at the beginning of each production. Thus, we can easily derive the manufacturer's objective (payoff) function Π_m :

$$\max_{q_l, T, \sigma_v, t_{vij}, z_{ij}, \tau_{ijk}} \Pi_m = \sum_{l=1}^L q_l D_l - \sum_{v=1}^V \delta_v \sigma_v - \sum_{i=1}^I \sum_{j=1}^{N_i} F_{ij} z_{ij} - \frac{T}{2} \sum_{l=1}^L \left(D_l \left(1 + \frac{1}{k_l} - \frac{D_l}{P_l} \right) h_{m_l} \right) - \frac{S}{T} - \gamma \quad (8)$$

Subject to

$$q_l = \omega_{m_l} - \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) - \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} - c_{m_l}, \quad (9)$$

$$\sum_{j=1}^{k-1} \tau_{ijk} + z_{ik} = 1, \quad \forall i=1,2,\dots,I; j=1,2,\dots,N_i-1; k=j+1,\dots,N_i \quad (10)$$

$$\tau_{ijk} \leq z_{ij}, \quad \forall i=1,2,\dots,I; j=1,2,\dots,N_i-1; k=j+1,\dots,N_i \quad (11)$$

$$t_{vij} = 0, \quad \forall L_{ij} \notin Q_v, \quad \forall v=1,2,\dots,V \quad (12)$$

$$\sum_{v=1}^V t_{vij} = z_{ij}, \quad \forall i=1,2,\dots,I; j=1,2,\dots,N_i \quad (13)$$

$$\sigma_v \leq \sum_{L_{ij} \in Q_v} t_{vij} \leq \Omega_v \sigma_v, \quad \forall v=1,2,\dots,V \quad (14)$$

$$z_{ij}, t_{vij}, \sigma_v, \tau_{ijk} = \{0,1\}, \quad \forall i=1,2,\dots,I; j=1,2,\dots,N_i-1; k=j+1,\dots,N_i \quad (15)$$

$$q_l \geq 0, \quad T > 0. \quad \forall l=1,2,\dots,L \quad (16)$$

Constraint (9) gives the relationship between the prices (the wholesale price and the component prices) and the manufacturer's profit margin. The amount of components used for unit product l consists of two parts. If the component used for product l is not the lowest functionality component of a SCS, it can be used and replace any other lower functionality

components. The usage amount of this component per unit product l is $z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right)$.

Otherwise, it can only be chosen for the product and thus, its usage amount is $z_{iN_i} u_{iN_i l}$. As indicated in the fourth point of the assumption in Section 3, constraint (10) ensures that a component is either used or replaced by certain higher functionality component, but not both. Constraint (11) makes sure that only procured components can be used to replace other components. (10) and (11) together ensure that the demands for all components are satisfied. Also, they met the one-way substitutability constraint which ensures that a higher functionality component can replace a lower functionality component but not vice versa. Constraint (12) sets the value of $t_{vij} = 0$ for all components $L_{ij} \notin Q_v$ for all the suppliers. Constraint (13) indicates that a component is procured from exactly one supplier. Constraint (14) sets the value of σ_v to 1, if supplier v supplies a component, and ensures that if supplier v is selected, at least one component will be supplied by him and the number of different types of components supplied by supplier v is no greater than Ω_v . The value ranges of all the variables are set by constraints (15) and (16).

4.2.3 The suppliers' model

Each supplier's problem is to determine an optimal decision vector x_{s_v} ($v=1,2,\dots,V$), including pricing decisions for the components supplied, to maximize his net profit. Table 3 gives the parameters and decision variables for the suppliers.

According to the given VMI policy, the suppliers do not pay for inventory cost. The supplier v faces component costs and an annual fixed cost. If a component supplied by supplier v is not the lowest functionality component of a SCS, its usage amount per unit

product l is $z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) t_{vij}$. Otherwise, the usage amount of the component is

$z_{iN_i} u_{iN_i l} t_{viN_i}$. Therefore, the supplier v 's objective (payoff) function Π_{s_v} is:

$$\max_{\omega_{ij}, L_{ij} \in Q_v} \Pi_{s_v} = \sum_{l=1}^L \sum_{i=1}^I \left(\sum_{j=1}^{N_l-1} (\omega_{ij} - c_{ij}) z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_l} u_{ikl} \tau_{ijk} \right) D_l t_{vij} + (\omega_{iN_l} - c_{iN_l}) z_{iN_l} u_{iN_l} D_l t_{viN_l} \right) - \eta_v \sigma_v \quad (17)$$

Subject to

$$\omega_{ij} \geq 0, \quad \forall L_{ij} \in Q_v \quad (18)$$

Constraint (18) ensures the non-negativeness of ω_{ij} . Here, we do not constraint ω_{ij} to be larger than its cost. That is because, the supplier may lower down the price for one component (even lower than its cost) to attract the manufacturer to buy his other components.

Table 3. Parameters and variables for suppliers

Ω_v :	Number of different types of components supplier v is capable of supplying
c_{ij} :	The cost of component L_{ij} paid by supplier
Q_v :	Set of components supplied by supplier v
η_v :	Supplier v 's annual fixed costs for the facilities and organization to carry the components
ω_{ij} :	Decision variable, the price of component L_{ij} charged by the supplier to the manufacturer

5 Model analysis and solution algorithm

Nash equilibrium is the most popular non-cooperative solution concept in game theory, which is widely used for dynamic non-cooperative game (Basar and Olsder, 1982). To obtain the Nash equilibrium, each player of the game takes the other players' decisions as given input parameters to determine his own decisions and adjusts his decisions corresponding to the changing of the other players' decisions to maximize his profit. The process continues until no player is willing to change his decisions, because any unilaterally changing will bring loss to him and then the Nash equilibrium is achieved.

In this paper, we apply Nash equilibrium concept to analyze the strategies of the supplier, the manufacturer and the retailers in the whole RMS game. Based on the analytical theory of Liu (1998), we first calculate the best reaction functions of each player and then employ algorithm procedure to build the Nash equilibrium.

5.1 Reaction functions

5.1.1 The retailers' reactions

We express retailer l 's demand function by the corresponding profit margins. Substituting (2) and (9), we can rewrite (5) as:

$$D_l = a_l - b_l \left(g_l + q_l + \sum_{i=1}^I \sum_{j=1}^{N_l-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_l} u_{ikl} \tau_{ijk} \right) + \sum_{i=1}^I \omega_{iN_l} z_{iN_l} u_{iN_l} + c_{m_l} \right). \quad (19)$$

Now suppose that the decision variables for the suppliers and the manufacturer are fixed. Then the retailer l 's problem of finding the optimal replenishment cycle becomes:

$$\min_{k_l} U_l = \frac{TD_l}{2k_l} h_l + \frac{k_l O_l}{T}. \quad (20)$$

The best reaction k_l that minimize U_l is by the smallest integer k_l^* that satisfies (Rosenblatt and Lee, 1985; Viswanathan and Wang, 2003):

$$k_l^* (k_l^* - 1) \leq \frac{T^2 h_l D_l}{2O_l} \leq k_l (k_l + 1) \quad \text{or} \quad (21)$$

$$k_l^* = \left\lfloor \left(1 + \sqrt{1 + \frac{2T^2 h_l D_l}{O_l}} \right) / 2 \right\rfloor. \quad (22)$$

Here, we define $\lfloor a \rfloor$ as the largest integer no larger than a .

We then consider the optimal value of g_l . From constraints (6) and (7), we can obtain lower bound and the upper bound of g_l :

$$\underline{g}_l = \max \left(0, \frac{a_l - P_l}{b_l} - \left(q_l + \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right) \right), \quad (23)$$

$$\overline{g}_l = \frac{a_l}{b_l} - \left(q_l + \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right). \quad (24)$$

Substituted (19) into (2), we can see that Π_{r_l} is a quadratic function of g_l . Because the second derivative of Π_{r_l} with respect to g_l is negative, we have:

$$\frac{\partial^2 \Pi_{r_l}}{\partial g_l^2} = -2b_l < 0. \quad (25)$$

Thus, Π_{r_l} is a concave function of g_l .

Set the first derivative of Π_{r_l} with respect to g_l equal to zero. Then g_l can be obtained as:

$$g_l = \frac{C_l}{2b_l} + \frac{h_l T}{4k_l}, \quad (26)$$

$$\text{where } C_l = a_l - b_l \left(q_l + \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right).$$

If g_l obtained from (26) is in the interval of $[\underline{g}_l, \overline{g}_l]$, it is obviously the optimal reaction g_l^* of the retailer. Otherwise, we have to substitute the bounds (23) and (24) into (2), the bound that provides higher profit is the best reaction g_l^* .

5.1.2 The manufacturer's reactions

Assume that the decision variables for the suppliers and the retailers are fixed. The manufacturer's problem of finding the optimal setup time interval in this case becomes:

$$\min_T U_m = \frac{T}{2} \sum_{l=1}^L \left(D_l \left(1 + \frac{1}{k_l} - \frac{D_l}{P_l} \right) h_{m_l} \right) + \frac{S}{T}. \quad (27)$$

Since the second derivative of (27), $\partial^2 U_m / \partial T^2 = 2S/T \geq 0$, the optimal T for the minimum of U_m can be derived from:

$$\frac{\partial U_m}{\partial T} = \frac{1}{2} \sum_{l=1}^L \left(D_l \left(1 + \frac{1}{k_l} - \frac{D_l}{P_l} \right) h_{m_l} \right) - \frac{S}{T^2} = 0 \quad \text{or} \quad (28)$$

$$T^* = \sqrt{\frac{2S}{\sum_{l=1}^L \left(D_l \left(1 + \frac{1}{k_l} - \frac{D_l}{P_l} \right) h_{m_l} \right)}}. \quad (29)$$

Obviously, the optimal T^* obtained from (29) satisfies constraint (16).

The net profit Π_m is the quadratic function of q_l . From constraints (7) and (16), we can obtain lower and the upper bounds of q_l :

$$\underline{q}_l = \max \left(0, \frac{a_l - P_l}{b_l} - \left(g_l + \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right) \right), \quad (30)$$

$$\overline{q}_l = \frac{a_l}{b_l} - \left(g_l + \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right). \quad (31)$$

Π_m 's second derivative about q_l is

$$\frac{\partial^2 \Pi_m}{\partial q_l^2} = -2b_l + \left(1 + \frac{1}{k_l} \right) b_l T + \frac{b_l^2 h_{m_l} T}{P_l}. \quad (32)$$

If $\partial^2 \Pi_m / \partial q_l^2 \leq 0$, the optimal q_l can be obtained from the first order condition of Π_m :

$$\frac{\partial \Pi_m}{\partial q_l} = D_l - b_l q_l - \frac{T}{2} \left((D_l - b_l) \left(1 + \frac{1}{k_l} \right) + \frac{2b_l D_l}{P_l} \right) h_{m_l} = 0. \quad (33)$$

Substitute (19) into (33), we have:

$$q_l = - \frac{W_l - \frac{h_{m_l} T}{2} \left(1 + \frac{1}{k_l} \right)}{2 - \frac{h_{m_l} T}{2} \left(1 + \frac{1}{k_l} + \frac{2b_l}{P_l} \right)} + W_l, \quad (34)$$

where $W_l = \frac{a_l}{b_l} - \left(g_l + \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right)$.

If q_l obtained from (34) is in the interval of $[\underline{q}_l, \overline{q}_l]$, it is the optimal reaction q_l^* of the manufacturer. Otherwise, Π_m reaches its maximal value when q_l is at its upper bound or lower bound. The bound that provides higher profit is the optimal reaction q_l^* .

If $\partial^2 \Pi_m / \partial q_l^2 > 0$, we also have to find the bound that provides maximal value of Π_m . That is the best reaction q_l^* .

Substitute (19) into the manufacturer's objective function (8), then the manufacturer's problem to find out optimal suppliers and components to maximize his profit is 0-1 integer programming with objection function (8) and Constraints (10)-(15). Some optimization software, such as Lingo, etc., can be used to solve this programming to obtain the best reactions τ_{ijk}^* , σ_v^* , t_{vij}^* , and z_{ij}^* .

5.1.3 The suppliers' reactions

Lastly, we consider the reaction functions for the suppliers. Suppose that the decision variables for the retailers and the manufacturer are fixed. The supplier's problem is to find out the optimal reaction for prices for the components his supplies. The second order condition

for Π_{s_v} in (17),

$$\frac{\partial^2 \Pi_{s_v}}{\partial \omega_{mn}^2} = \begin{cases} -2 \sum_{l=1}^L b_l z_{mn}^2 \left(u_{mnl} + \sum_{k=n+1}^{N_i} u_{mkl} \tau_{mnk} \right)^2 t_{vmn}, & 1 \leq n \leq N_i - 1 \\ -\sum_{l=1}^L 2b_l z_{mN_i}^2 u_{mN_i l}^2 t_{vmN_i}, & n = N_i \end{cases}. \quad (35)$$

Obviously, $\partial^2 \Pi_{s_v} / \partial \omega_{mn}^2 \leq 0$.

Thus, the necessary condition to maximize the supplier's net profit Π_{s_v} is:

$$\frac{\partial \Pi_{s_v}}{\partial \omega_{mn}} = \begin{cases} \sum_{l=1}^L z_{mn} \left(u_{mnl} + \sum_{k=n+1}^{N_m} u_{mkl} \tau_{mnk} \right) (D_l t_{vmn} - b_l \Gamma_l) = 0, & 1 \leq n \leq N_m - 1 \\ \sum_{l=1}^L z_{mN_m} u_{mN_m l} (D_l t_{vmN_m} - b_l \Gamma_l) = 0, & n = N_m \end{cases} \quad (36)$$

$$\text{where } \Gamma_l = \sum_{i=1}^I \left(\sum_{j=1}^{N_i-1} (\omega_{ij} - c_{ij}) t_{vij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + (\omega_{iN_i} - c_{iN_i}) t_{viN_i} z_{iN_i} u_{iN_i l} \right).$$

Substitute (19) into (36), we can obtain:

$$\omega_{mn} = \begin{cases} \frac{\sum_{l=1}^L \left(u_{mnl} + \sum_{k=n+1}^{N_m} u_{mkl} \tau_{mnk} \right) (B_{mnl} t_{vmn} - b_l \Gamma_{mnl})}{\sum_{l=1}^L 2b_l z_{mn} \left(u_{mnl} + \sum_{k=n+1}^{N_m} u_{mkl} \tau_{mnk} \right)^2 t_{vmn}}, & 1 \leq n \leq N_m - 1 \\ \frac{\sum_{l=1}^L u_{mN_m l} (B_{mN_m l} t_{vmN_m} - b_l \Gamma_{mN_m l})}{\sum_{l=1}^L 2b_l z_{mN_m} u_{mN_m l}^2 t_{vmN_m}}, & n = N_m \end{cases}, \quad (37)$$

where

$$B_{mnl} = a_l - b_l \left(g_l + q_l + \sum_{\substack{i=1 \\ i \neq m}}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{\substack{j=1 \\ j \neq n}}^{N_m-1} \omega_{mj} z_{mj} \left(u_{mj l} + \sum_{k=j+1}^{N_m} u_{mkl} \tau_{mjk} \right) + \sum_{i=1}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right),$$

$$\Gamma_{mnl} = \sum_{\substack{i=1 \\ i \neq m}}^I \sum_{j=1}^{N_i-1} (\omega_{ij} - c_{ij}) t_{vij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{\substack{j=1 \\ j \neq n}}^{N_m-1} (\omega_{mj} - c_{mj}) t_{vmj} z_{mj} \left(u_{mj l} + \sum_{k=j+1}^{N_m} u_{mkl} \tau_{mjk} \right) \\ - t_{vmn} z_{mn} c_{mn} \left(u_{mnl} + \sum_{k=n+1}^{N_m} u_{mkl} \tau_{mnk} \right) + \sum_{i=1}^I (\omega_{iN_i} - c_{iN_i}) t_{viN_i} z_{iN_i} u_{iN_i l}$$

$$B_{mN_m l} = a_l - b_l \left(g_l + q_l + \sum_{i=1}^I \sum_{j=1}^{N_i-1} \omega_{ij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{\substack{i=1 \\ i \neq m}}^I \omega_{iN_i} z_{iN_i} u_{iN_i l} + c_{m_l} \right),$$

$$\Gamma_{mN_m l} = \sum_{i=1}^I \sum_{j=1}^{N_i-1} (\omega_{ij} - c_{ij}) t_{vij} z_{ij} \left(u_{ijl} + \sum_{k=j+1}^{N_i} u_{ikl} \tau_{ijk} \right) + \sum_{\substack{i=1 \\ i \neq m}}^I (\omega_{iN_i} - c_{iN_i}) t_{viN_i} z_{iN_i} u_{iN_i l} - c_{mN_m} t_{vmN_m} z_{mN_m} u_{mN_m l}.$$

From constraints (7) and (18), we can obtain the upper bound and lower bound of

ω_{mn} :

$$\underline{\omega}_{mn} = \begin{cases} \max \left(0, \max_l \frac{B_{mnl} - P_l}{b_l z_{mn} \left(u_{mnl} + \sum_{k=n+1}^{N_m} u_{mkl} \tau_{mnk} \right)} \right), & 1 \leq n \leq N_m - 1 \\ \max \left(0, \max_l \frac{B_{mN_m l} - P_l}{b_l z_{mN_m} u_{mN_m l}} \right), & n = N_m \end{cases}, \quad (38)$$

$$\overline{\omega}_{mn} = \begin{cases} \min_l \frac{B_{mnl}}{b_l z_{mn} \left(u_{mnl} + \sum_{k=n+1}^{N_m} u_{mkl} \tau_{mnk} \right)}, & 1 \leq n \leq N_m - 1 \\ \min_l \frac{B_{mN_m l}}{b_l z_{mN_m} u_{mN_m l}}, & n = N_m \end{cases}. \quad (39)$$

If ω_{mn} obtained from (37) is in the interval of $[\underline{\omega}_{mn}, \overline{\omega}_{mn}]$, it is the best reaction ω_{mn}^* of supplier v . Otherwise, we have to substitute the bounds (38) and (39) into (12), the bound that provides higher profit is the optimal reaction ω_{mn}^* .

5.2 Algorithm for the Nash equilibrium of RMS game

In this sub-section, we give the following solution algorithm to compute the equilibrium of the three-level dynamic non-cooperative game. As Section 4.1, X_{λ_w} is denoted as the strategy set of the supply chain member λ_w , $\lambda_w \in \{s_1, \dots, s_v, m, r_1, \dots, r_L\}$. $X_\lambda = X_{\lambda_1} \times \dots \times X_{\lambda_w}$ is the strategy profile set of supply chain members λ_1 to λ_w . Thus, X_s , X_r , X_{sm} and X are the strategy profile sets of all the suppliers, all the retailers, the suppliers and the manufacturer, and all the chain members, respectively. x_i is the strategy / strategy profile of strategy set / strategy profile set X_i . Let x_{-i} be the strategy profile of all the supply chain members except for i . We present the following algorithm for solving the three-level dynamic non-cooperative game model:

Step 0: Give the initial strategy profile for all suppliers, the manufacturer and all retailers $x^{(0)} = ((x_s^{(0)}, x_m^{(0)}), x_r^{(0)})$ in the strategy profile set X .

Step 1: For each retailer l , based on $x_{-r_l}^{(0)}$, the optimal reaction $x_{r_l}^* = (k_l^*, g_l^*)$ is obtained as Section 5.1.1 in strategy set X_{r_l} .

Step 2: Fixed $x_{-m}^{(0)}$, based on $x_m^{(0)}$, find out the optimal reaction $x_m^* = (T^*, q_l^*, \tau_{ijk}^*, \sigma_v^*, t_{vij}^*, z_{ij}^*)$ as Section 5.1.2 in strategy set X_m .

Step 3: For each supplier v , based on $x_{-s_v}^{(0)}$, find out the optimal reaction $x_{s_v}^* = (\omega_{mn}^*)$, for all $L_{mn} \in Q_v$, as Section 5.1.3. If $\|x_s^* - x_s^{(0)}\| \leq \varepsilon_1^{(1)}$, the Nash equilibrium of SS game, x_s^* , obtained, Go step 4.

Step 4: $x_{sm}^* = (x_s^*, x_m^*)$. If $\|x_{sm}^* - x_{sm}^{(0)}\| \leq \varepsilon_2^{(1)}$, the Nash equilibrium of MS game, x_{ms}^* , obtained, Go step 5.

Step 5: $x^* = (x_{sm}^*, x_r^*)$. If $\|x^* - x^{(0)}\| \leq \varepsilon_3^{(1)}$, the Nash equilibrium of RMS game, x^* , obtained.

Output the optimal results and stop. Otherwise, $x^{(0)} = x^*$, go step 1.

(① $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are very small positive numbers)

6 Numerical results

To demonstrate the application of the proposed game model and solution algorithm, we present and discuss a specific case in a computer industry. This particular case is concerned with coordinating suppliers and components selection, pricing and replenishment decisions of the supply chain (as Fig. 1) described in Section 3. The explicit numerical parameters selected for the base case example reflect those shown in Lu (1995), Woo et al. (2001), Prafulla et al. (2006). For example, the holding cost per unit final product at any retailer should be higher than the manufacturer's. The manufacturer's setup cost should be much larger than any ordering cost. All the parameters are given in Table 5 and the relationship between the components and their suppliers is shown as Fig. 1.

To investigate the influence of varying parameter and integrating product family design, we conduct sensitive analysis on market scale parameter and for different values of market scale and consider two cases, i.e. no components selection (NCS) case and with components selection (WCS) case. For the first one, components used for each product have already been fixed a priori. No substitution could be conducted. For the second one, although components used for each product have been fixed, higher functionality components can be used to substitute lower ones. By applying the solution procedure in section 5.2, the optimal decisions and profits for the suppliers, the manufacturer and the retailers are summarized in Table 4 and Table 5.

6.1 Simulation results

Table 4 illustrates the optimal suppliers and components selection, the components pricing decisions and profits for NCS case (Table 4(a)) and WCS case (Table 4(b)) under different values of market scale.

As can be seen, when a_1 is 2.0×10^6 , the same suppliers and components are selected for NCS case and WCS case. However, we observe from Table 4(b) that when a_1 increases to 2.5×10^6 , component L_{23} is substituted by L_{22} for product 2. Moreover, when a_1 is 5.0×10^6 , the highest functionality components L_{11} and L_{21} are used to substitute lower functionality components L_{12} and L_{22} for both products. On this occasion, supplier 2 and supplier 3 are not selected, while supplier 4 is selected. This shows that in WCS case, less suppliers are selected and higher functionality components are used to replace lower ones as a_1 increases.

Table 4 also exhibits that higher values of a_1 increase the total profit of the suppliers in both NCS case and WCS case. Yet, the total profit of the suppliers in NCS case is higher than WCS case and the profit disparity between NCS case and WCS case is increasing in a_1 . For example, when a_1 is 2.5×10^6 , the suppliers' total profit in NCS case (3.9867×10^8) is only 7.27% greater than that in WCS case (3.7166×10^8). However, when a_1 increases to 5.0×10^6 , the profit gap is widen to 64.83%. And supplier s_1 , who provides components that only used for product 1, has a most profit increase. In the above case, when a_1 increases to 2.5×10^6 , s_1 's profit increases from 1.9729 to 4.5474, by 130.49%, while the other suppliers'

profit increases are no more than 38.32%.

The optimal decisions and profits of the manufacturer and the retailers for both NCS case and WCS case by varying the market scale parameter a_1 are illustrated in Table 5. It can be seen that both the wholesale prices and the retail prices for the two products are increasing in a_1 and product 1's prices increase more significantly. We also find that as a_1 increases, the manufacturer's profit, retailer 1's profit and the total profits of the supply chain members (i.e. system profit) improve, whereas retailer 2's profit decreases.

Comparing the two cases, we can see that the wholesale price and the retail price for product 1 are lower in WCS case, while the prices for product 2 are higher. In contrast, in WCS case, the profits for the manufacturer and retailer 1 are higher, and retailer 2's profit and the system profit are lower. Moreover, considering when a_1 increases from 2.0×10^6 to 3.5×10^6 , the manufacturer's profit and retailer 1's profit in WCS case are 17.32% and 4.84% larger than the profits in NCS case. When a_1 increases to 5.0×10^6 , the profit gaps are widen to 66.95% and 13.78%, respectively. That is the manufacturer and retailer 1 benefit more in WCS case as a_1 increases. However, Table 5 also shows that the system profit increases more significantly in NCS case and for retailer 2's, its profit decreases more significantly in WCS case.

From Table 5, we also observe that a larger value of a_1 shortens the setup time interval for retailer 1 and retailer 1 will replenish his product more frequently in WCS case than in NCS case. Observed from NCS case, when a_1 increases from 3.5×10^6 to 5.0×10^6 , the cycles for retailer 1 reduce from 0.0356/12 to 0.0332/14. And the replenishment cycles of retailer 1 in WCS case are 0.0358/13, 0.1195/52, which are shorter than 0.0356/12, 0.0332/14 in NCS case.

6.2 Managerial implications

Based on the results obtained from the simulation results presented above, in this subsection, we wrap up our findings and provide some important managerial insights.

The first finding is that an increase in one retailer's market scale would increase this retailer's and the manufacturer's profits, as well as the suppliers' total profit and the system profit, while decreasing the other retailer's profit. Specially, such benefit for the suppliers mostly goes to the suppliers who only provide components for the product in this market. When one retailer's market scale increases, the manufacturer would rather provide more product to this retailer and cut the production for other product to achieve higher profit. The increased demand for this product brings larger profits for the supply chain system and the corresponding components suppliers.

Secondly, in the presence of the component selection strategy, when one retailer's market scale is large, the manufacturer tends to use higher functionality components for the product in this market. Consequently, the manufacturer could benefit from the component selection strategy, and the supply chain system includes fewer components and less suppliers and their total profit becomes lower. However, when the market scale is low, the manufacturer would build up the product exactly the same as the retailer requires.

It is also worth mentioning that as one retailer's market scale increases, the manufacturer and this retailer could benefit more from the components selection strategy. In the contrary, the suppliers as a group, the other retailer and the supply chain system benefit more with the absence of this strategy. That is, in NCS case, the total profit of the suppliers and the system profit increase more significantly. The profits for the manufacturer and this retailer increase more significantly and the other retailer's profit decreases more significantly in WCS case.

Finally, when one retailer's market scale is large, the components selection strategy would increase this retailer's order frequency. The components selection strategy would enhance the demand for this retailer. Thus, a shorter replenishment cycle is required.

Table 4. Decisions and profits of the suppliers

(a)NCS case

<i>Supplier & Supplied components</i>		<i>s₁</i>		<i>s₂</i>			<i>s₃</i>		<i>s₄</i>		<i>s₅</i>	<i>s₆</i>		<i>Total profit</i>
		<i>L₁₁</i>	<i>L₂₁</i>	<i>L₁₂</i>	<i>L₂₃</i>	<i>L₆₁</i>	<i>L₂₂</i>	<i>L₄₁</i>	<i>L₃₁</i>	<i>L₄₁</i>	<i>L₅₁</i>	<i>L₃₁</i>	<i>L₆₁</i>	
<i>a₁</i>	<i>Results</i>													
2.0×10^6	<i>Price</i>	138.47		137.93	88.46		129.47	68.46			261.29	85.52	15.76	
	<i>Profit ($\times 10^7$)</i>	1.9729			6.6354		6.6349		0		6.3930	6.3924		28.0286
2.5×10^6	<i>Price</i>	168.98		131.16	118.97		92.21	98.96			274.59	114.59	4.59×10^{-13}	
	<i>Profit ($\times 10^7$)</i>	4.5474			8.8178		8.8173		0		8.8424	8.8418		39.8667
3.5×10^6	<i>Price</i>	229.96		117.67	179.96		17.74	159.94			301.20	141.20	7.78×10^{-13}	
	<i>Profit ($\times 10^7$)</i>	12.372			15.878		15.877		0		14.878	14.877		73.882
5.0×10^6	<i>Price</i>	333.10		81.54	283.09		0	206.99			340.06	180.06	3.97×10^{-13}	
	<i>Profit ($\times 10^7$)</i>	32.614			34.179		29.578		0		26.550	26.549		149.47

(b) WCS case

<i>Supplier & Supplied components</i>		<i>s₁</i>		<i>s₂</i>			<i>s₃</i>		<i>s₄</i>		<i>s₅</i>	<i>s₆</i>		<i>Total profit</i>
		<i>L₁₁</i>	<i>L₂₁</i>	<i>L₁₂</i>	<i>L₂₃</i>	<i>L₆₁</i>	<i>L₂₂</i>	<i>L₄₁</i>	<i>L₃₁</i>	<i>L₄₁</i>	<i>L₅₁</i>	<i>L₃₁</i>	<i>L₆₁</i>	
<i>a₁</i>	<i>Results</i>													
2.0×10^6	<i>Price</i>	138.47		137.93	88.46		129.47	68.46			261.29	85.52	15.76	
	<i>Profit ($\times 10^7$)</i>	1.9729			6.6354		6.6349		0		6.3930	6.3924		28.0286
2.5×10^6	<i>Price</i>	176.63		129.82			93.66	94.44			278.09	75.13	42.96	
	<i>Profit ($\times 10^7$)</i>	5.0930			4.1937		9.2937		0		9.2931	8.8418		37.1659
3.5×10^6	<i>Price</i>	264.10		97.28			71.81	148.37			310.17	52.63	97.53	
	<i>Profit ($\times 10^7$)</i>	16.014			2.3916		16.325		0		16.325	14.877		67.3796
5.0×10^6	<i>Price</i>	207.91	213.26							155.17	321.17	75.13	83.32	
	<i>Profit ($\times 10^7$)</i>	22.671			0		0		22.671		22.671	22.671		90.685

implies the component is not selected for the platform products

a_1	Manufacturer & Retailer Results	NCS case				WCS case			
		m		r_1	r_2	m		r_1	r_2
		Product 1	Product 2			Product 1	Product 2		
2.0×10^6	Price	695.83	782.92	1,062.20	1,224.80	695.83	782.92	1,062.20	1,224.80
	Cycle	0.0411	0.0411	0.0411/10	0.0411/17	0.0411	0.0411	0.0411/10	0.0411/17
	Demand($\times 10^5$)	5.1291	5.3025	5.1291	5.3025	5.1291	5.3025	5.1291	5.3025
	Profit($\times 10^7$)	5.6451		18.791	23.430	5.6451		18.791	23.430
	System profit($\times 10^8$)	7.5895				7.5895			
2.5×10^6	Price	843.90	789.68	1,314.80	1,228.20	836.24	791.03	1,311.00	1,228.80
	Cycle	0.0389	0.0389	0.0389/11	0.0389/16	0.0384	0.0384	0.0384/11	0.0384/16
	Demand($\times 10^5$)	6.5927	5.2619	6.5927	5.2619	6.6463	5.2538	6.6463	5.2538
	Profit($\times 10^7$)	7.6617		31.045	23.073	8.1211		31.552	23.002
	System profit($\times 10^8$)	10.1646				9.9841			
3.5×10^6	Price	1,140.10	803.17	1,820.0	1,234.90	1,105.90	823.57	1,803.00	1,245.10
	Cycle	0.0356	0.0356	0.0356/12	0.0356/15	0.0358	0.0358	0.0358/13	0.0358/15
	Demand($\times 10^5$)	9.5196	5.1810	9.5196	5.1810	9.7586	5.0586	9.7586	5.0586
	Profit($\times 10^7$)	14.342		64.731	22.369	16.826		68.021	21.324
	System profit($\times 10^8$)	17.5324				17.3551			
5.0×10^6	Price	1,572.60	839.31	2,572.00	1,253.00	1,418.80	1,059.70	2,495.10	1,363.20
	Cycle	0.0332	0.0332	0.0332/14	0.0332/14	0.1195	0.1195	0.1195/52	0.1195/42
	Demand($\times 10^5$)	13.992	4.9642	13.992	4.9642	15.068	3.6419	15.068	3.6419
	Profit($\times 10^7$)	31.961		139.83	20.536	53.360		162.18	11.053
	System profit($\times 10^8$)	34.1797				31.7278			

Parameters for base example: $a_1 = a_2 = 2 \times 10^6$; $b_1 = 1400$, $b_2 = 1200$; $O_1 = 4$, $O_2 = 3$; $h_1 = 1$, $h_2 = 2$, $S = 100$, $h_{m_1} = 0.5$, $h_{m_2} = 1$, $P_1 = 5 \times 10^6$, $P_2 = 3 \times 10^6$, $c_{m_1} = 10$, $c_{m_2} = 5$, $\delta_1 = 12$, $\delta_2 = 13$, $\delta_3 = 40$, $\delta_4 = 60$, $\delta_5 = 18$, $\delta_6 = 31$, $F_{11} = 3$, $F_{12} = 4$, $F_{21} = 6$, $F_{22} = 7$, $F_{23} = 8$, $F_{31} = 9$, $F_{41} = 12$, $F_{51} = 2$, $F_{61} = 10$, $c_{L_{41}} = 100$, $c_{L_{42}} = 50$, $c_{L_{21}} = 200$, $c_{L_{22}} = 80$, $c_{L_{23}} = 50$, $c_{L_{31}} = 20$, $c_{L_{41}} = 30$, $c_{L_{51}} = 200$, $c_{L_{61}} = 29$, $R_{s_1} = 30$, $R_{s_2} = 15$, $R_{s_3} = 35$, $R_{s_4} = 23$, $R_{s_5} = 20$, $R_{s_6} = 18$, $R_m = 100$, $R_{\eta_1} = 50$, $R_{\eta_2} = 61$.

7 Conclusion

In this paper, we have considered the coordination of suppliers and components selection, pricing, and replenishment decisions in a multi-level supply chain composed of multiple suppliers, one single manufacturer and multiple retailers. This coordination problem is modeled as a three-level dynamic non-cooperative game model. We use both analytical and computational methods for the derivative of the optimal decisions of all the chain members. A numerical study is conducted to examine the game model and solution algorithm. The numerical results show that the increase of one retailer's market scale will decrease the other retailer's profit and shorten his own replenishment cycle. Moreover, when one retailer's market scale is large and a components selection strategy is simultaneously employed, the manufacturer tends to use high-end components to substitute the low-end ones for the product sold to this retailer and the manufacturer would generally benefit from this strategy. However, the supply chain system may become leaner with a fewer number of components and suppliers and their total profit would be lower. Individually, some suppliers may also benefit from this components selection strategy while others may suffer from a loss in profits.

The contributions of the paper to the literature are as follows. Most literature to date have focused on pricing and inventory coordination in the two-echelon channel. This paper is an important addition to the literature on coordinating pricing and inventory decisions in a multi-level supply chain. Furthermore, this paper incorporates product family design with pricing and inventory coordination problem. In such a situation, products can be built more flexibly with the market. Lastly, significantly different from most extant literature in supply chain coordination which regard suppliers and components are selected already, we consider supplier selection and component selection as decision variables and determine them through a whole supply chain dynamic game.

This paper has several limitations which can be extended in the further research. The competition among multiple products and among multiple retailers is not covered in this paper. Under this competition, the demand of one product / retailer is not only the function of his own price, but also the other products' / retailers' prices. Secondly, we assume that the components can be ranked in the order of decreasing functionality, and either all or none the demand of a component is replaced by the lower order component in the same SCS. Future research should relax these constraints and include the case that the components can be partially replaced by higher functionality components. Also, we assume that the production rate is greater than or equal to the demand rate to avoid shortage cost. Without this assumption, the extra cost should be incorporated into the future work.

Acknowledgments

The authors would like to thank the referees and editor-in-chief for their valuable comments that have helped improve this paper.

Reference

- Abad, P.L., 2003. Optimal price and lot size when the supplier offers a temporary price reduction over an interval. *Computers and Operations Research* 30(1), 63-74.
- Arcelus, F.J., G. Srinivasan, 1987. Inventory policies under various optimizing criteria and variable markup rate. *Management Science* 33(6), 756-762.
- Ardalan, A., 1991. Combined optimal price and optimal inventory replenishment policies

- when a sale results in increase in demand. *Computers and Operations Research* 18(9), 721-730.
- Bakos, JY, Brynjolfsson, E., 1993. Information technology, incentives, and the optimal number of suppliers. *Journal of Management Information Systems* 10(2), 37-53.
- Basar T., Olsder G.J., 1982. *Dynamic noncooperative Game Theory*. Academic Press.
- Birendra K. Mishra, Srinivasan Raghunathan, 2004. Retailer- vs. vendor-managed inventory and brand competition. *Management Science* 50(4), 445-457.
- Boyaci, T., G. Gallego, 2002. Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. *International Journal of Production Economics* 77(2), 95-111.
- Cachon G, Netessine, S, 2004. Game theory in supply chain analysis, in *supply chain analysis in the e-business era*. E.b.D. Simchi-Levi, S. D. Wu, and M. Shen, Eds. Norwell, MA: Kluwer Academic, Chapter 2.
- Chakravarty, A. K., Baum, J. E., 1992. Coordinated planning for competitive products and their manufacturing operations. *International Journal of Production Research* 30(10), 2293-2311.
- Chan, LMA, ZJM Shen, D Simchi-Levi, JL Swann, 2004. Coordination of pricing and inventory decisions: A survey and classification. *International Series in Operations Research and Management*, Chapter 14.
- Collier, D. A, 1981. The measurement and operating benefits of component part commonality. *Decision Science* 12(1), 85-96.
- Dutta, P.K., 1999. *Strategies and games: Theory and practice*. The MIT Press.
- Esmaili, M., Mir-Bahador Aryanezhad, P. Zeepongsekul, 2009. A game theory approach in seller-buyer supply chain. *European Journal of Operations Research* 191(2), 442-448.
- Graves, SC, Willems, SP, 2005. Optimizing the supply chain configuration for new products. *Management Science* 51(8), 1165-1180.
- Huang G. Q., Xin Yan Zhang, Victor H. Y. Lo, 2007. Integrated configuration of platform products and supply chains for mass customization: a game-theoretic approach. *IEEE Transactions on Engineering Management* 54(1), 156-171.
- Jeuland A. P., Shugan S. M., 1988. Channel of distribution profits when channel members form conjectures. *Marketing Science* 7(2), 202-210.
- Kim, DaeSoo, Lee, W.J., 1998. Optimal joint pricing and lot sizing with fixed and variable capacity. *European Journal of Operational Research* 109(1), 212-227.
- Kohli, R., Sukumar, R., 1990. Heuristics for product-line design using conjoint analysis. *Management Science* 36 (12), 1464-1478.
- Krishnan, V., Singh, R., Tirupati, D., 1998. A model-based approach for planning and developing a family of technology-based products. Austin, The University of Texas at Austin Management Department, Working Paper.
- Kunreuther, H., J. F. Richard, 1971. Optimal pricing and inventory decisions for nonseasonal item. *Econometrica* 39(1), 173-175.
- Lariviere M.A., E.L. Porteus, 2001. Selling to the newsvendor: an analysis of price-only contracts. *Manuf. Service Operations Management* 3, 293-305.
- Liu, Baoding, 1998. Stackelberg-Nash equilibrium for multilevel programming with multiple followers using genetic algorithms. *Computer Math. Application* 36(7), 79-89.
- Lu, L., 1995. A one-vendor multi-buyer integrated inventory model. *European Journal of Operational Research* 81(2), 312-323.
- Macduffie, J. P., K. Sethuraman, M. L. Fisher, 1996. Product variety and manufacturing performance: evidence from the international automotive industry. *Management Science* 42(3), 350-369
- Martin, GE, 1994. Note on an EOQ model with a temporary sale price. *International Journal*

- of Production Economics 37(2-3), 241-243.
- McGuire, Timothy W. and Richard Staelin, 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science* 2(2), 161-190.
- Meyer, MH, Utterback, JM, 1993. The product family and the dynamics of core capability. *Sloan Management Review*.
- Meyer, M., Lehnerd, A. P., 1997. The power of product platform building value and cost leadership. New York: Free Press
- Moutaz Khouja, 2003. Optimizing inventory decisions in a multi-stage multi-customer supply chain. *Transportation Research Part E: Logistics and Transportation Review*, 39(3), 193-208.
- Moyaux, T., B. Chaib-draa, S. D'Amours, 2004. The impact of information sharing on the efficiency of an ordering approach in reducing the bullwhip effect. *IEEE Trans. on Systems, Man, and Cybernetics, Part C (SMC-C)*, 37(3), 396-409.
- Park, J., Simpson, TW, 2005. Development of a production cost estimation framework to support product family design. *International Journal of Production Research* 43(4), 731-772.
- Parlar, M. and Wang, Q., 1994. Discounting decisions in a supplier buyer relationship with a linear buyer's demand. *IIE Transactions* 26(2), 34-41.
- Porter, M., 1985. *Competitive advantage: creating and sustaining superior performance*, New York: The Free Press.
- Prafulla Joglekar, Madjid Tavana, Jack Rappaport 2006. A comprehensive set of models of intra and inter-organizational coordination for marketing and inventory decisions in a supply chain. *International Journal of Integrated Supply Management*.
- Rosenblatt, M.J., Lee, H.L., 1985. Improving profitability with quantity discounts under fixed demand. *IIE Transactions*, 17(4), 388-395.
- Sanderson, S., Mustafa Uzumeri, 1995. Managing product families: the case of the Sony walkman. *Research Policy*, 24(5), 761-782.
- Simchi-Livi, D., Kaminsky, P., Simchi-Livi, E., 2000. Designing and managing the supply chain-concepts. In: *Strategies and Case Studies*. McGraw-Hill, Singapore.
- Simpson, TW, 2005. *Product platform design and customization: Status and promise*, Cambridge Univ. Press.
- Slikker, M., J. Fransoo, and Wouters, 2005. Cooperation between multiple news-vendors with transshipments. *European Journal of Operational Research* 167(2), 370-380.
- Saurabh and Krishnan, 1999. Integrated component and supplier selection for a product family. *Production and Operations Management* 8(2), 163-182.
- Tersine, RJ, Price, RL, 1981. Temporary price discounts and EOQ. *Journal of Purchasing and Materials Management* 17(4), 23-27.
- Thonemann, UW, Bradley, J.R., 2002. The effect of product variety on supply-chain performance. *European Journal of Operational Research* 143(3) 548-556.
- Tullous, R., Utrecht, RL, 1992. Multiple or single sourcing. *Journal of Business and Industrial Marketing* 7(3), 5-18.
- Tyan, J., Wee, HM, 2003. Vendor managed inventory: A survey of the Taiwanese grocery industry. *Journal of Purchasing and Supply Management* 9(1), 11-18.
- Viswanathan S., Wang, Q., 2003. Discount pricing decisions in distribution channels with price-sensitive demand. *European Journal of Operational Research* 149(3), 571-587.
- Wang, Q. and Wu, Z., 2000. Improving a suppliers quantity discount gain from many different buyers. *IIE Transactions* 32(11), 1071-1079.
- Weng, ZK, 1995. Channel coordination and quantity discounts. *Management Science* 41(9), 1509-1522.
- Weng, ZK, Wong, RT, 1993. General models for the supplier's all-unit quantity discount

- policy. *Naval Research Logistics* 40(7), 971-991.
- Weng, K., 1997. Pricing and ordering strategies in manufacturing and distribution alliances. *IIE Transactions* 29(8), 681-692.
- Whitin, TM, 1955. Inventory control and price theory. *Management Science* 2(1), 61-68.
- Yano, C., Dobson, G., 1998. Profit-optimizing product line design, selection and pricing with manufacturing cost consideration. *Product Variety Management: Research Advances*, E. b. T. H. Ho and C. S. Tang, Eds. Norwell, MA: Kluwer Academic.
- Woo Y. Y., Shu-Lu Hsu, Soushan Wu, 2001. An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction. *International Journal of Production Economics*, 73(3), 203-215.
- Yu Yugang, Liang Liang, George Q. Huang, 2006. Leader-follower game in vendor-managed inventory system with limited production capacity considering wholesale and retail prices. *International Journal of Logistics: Research and Applications* 9(4), 335-350.
- Zhang XY, 2006. Simultaneous configuration of platform products and supply chains. Ph.D. Dissertation, Department of Industrial and Manufacturing Systems Engineering, The University of Hong Kong, China.